Edexcel Physics A-level

Topic 13: Oscillations

Notes

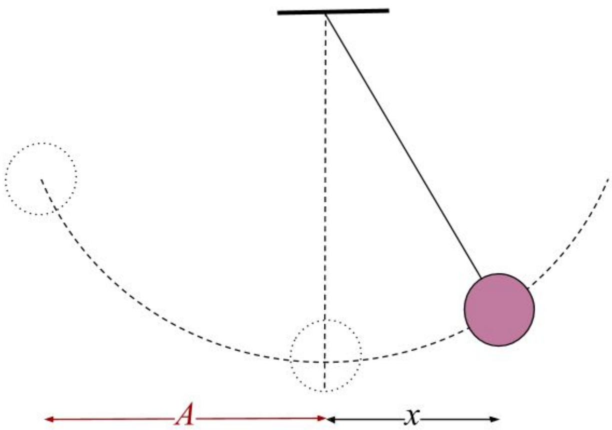
13 - Oscillations

13.181 - Simple harmonic motion

An object experiencing **simple harmonic motion** is one which experiences a **restoring force**, which acts towards the centre of equilibrium. This force is directly proportional to the object's distance from the equilibrium position and can be described using the equation below:

$$F = -kx$$

Where **F** is the restoring force, **k** is a constant which depends on the oscillating system and **x** is the distance from the equilibrium position.



An example of a simple harmonic oscillator is the **simple pendulum**, as shown in the diagram above. The pendulum oscillates around a central midpoint known as the equilibrium position. Marked on the diagram by an x is the measure of **displacement**, and by an A is the **amplitude** of the oscillations, this is the maximum displacement. You could also measure the **time period (T)** of the oscillations by measuring the time taken by the pendulum to move from the equilibrium position, to the maximum displacement to the left, then to the maximum displacement to the right and back to the equilibrium position.

In a simple pendulum, the **restoring force** is provided by the horizontal component of gravity acting on the pendulum bob.

There are many other examples of simple harmonic motion (SHM), all of which obey the condition for SHM, which is the existence of the restoring force as described above.

13.182 - Simple harmonic motion calculations

The acceleration of an object experiencing **simple harmonic motion** is **directly proportional** to **displacement** and is in the **opposite direction**. This can be shown through the equation:

$$a = -\omega^2 x$$

Where **a** is acceleration, **ω** is angular speed, **x** is displacement from the equilibrium position.

Angular speed (ω) is the angle an object moves through per unit time and can be calculated using the following formula:

$$\omega = 2\pi f$$

Where **f** is the frequency.

By rearranging the above formula so that its subject is frequency, you can derive the following formula for the time period of oscillations (T):

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

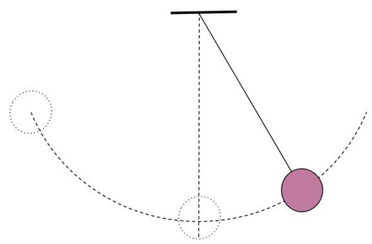
Using the measurements described in the section above, you can use the following formulas with simple harmonic oscillators:

$$x = A\cos \omega t$$
$$v = -A\omega\sin \omega t$$
$$a = -A\omega^2\cos \omega t$$

Where **A** is the amplitude of oscillation, **ω** is angular speed, **x** is displacement from the equilibrium position, **v** is the velocity and **a** is the acceleration.

13.183 - Calculating the time period

Simple harmonic systems are those which oscillate with simple harmonic motion, examples include:



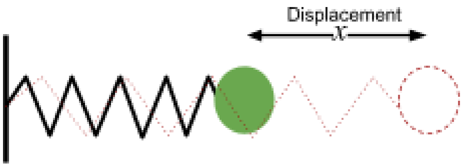
- **Simple pendulum** - A small, dense bob of mass m hangs from a string of length l , which is attached to a fixed point. For this type of system, you can use the following formula:

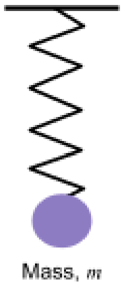
$$T = 2\pi\sqrt{\frac{l}{g}}$$

where T is time period, l is the length of the string, g is acceleration due to gravity

- **Mass-spring system** - There are two types of mass-spring system, where the spring is either vertical or horizontal, the only difference between these two is the type of energy which is transferred during oscillations.For either of these types of system you can use the following formula:

$T = 2\pi\sqrt{\frac{m}{k}}$ where T is time period, m is the mass, k is the spring constant



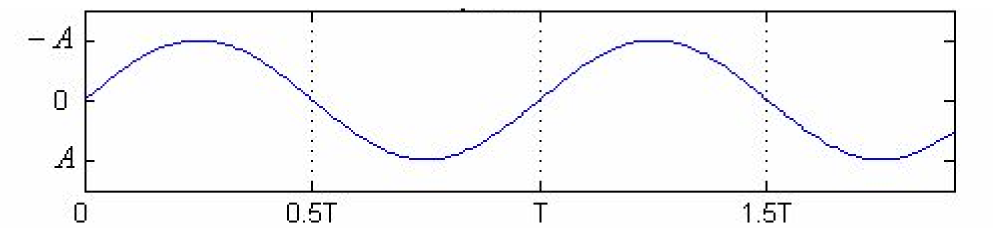


13.184 - Displacement-time graph for an oscillating system

You can represent the **displacement** from the equilibrium position (x) of an oscillating object over time by using a graph.

$$x = A\cos \omega t$$

By looking at the equation for displacement described earlier and also written above, you can see that a displacement-time graph will follow a **cosine** or a **sine** curve, with a maximum A, and minimum -A because A and **ω** are constants:



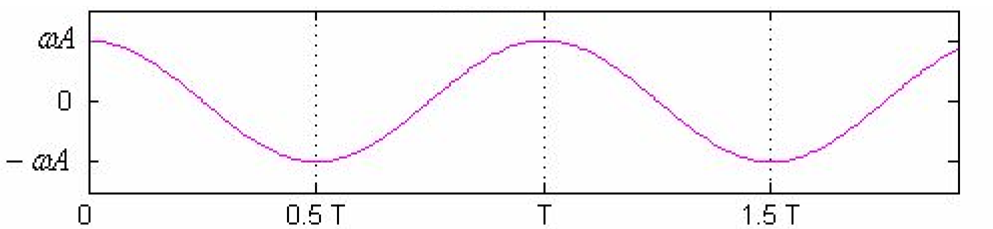
As **velocity** is the change in displacement over time, you can find the velocity of the system at a point, by finding the **gradient** of the curve at that point.

13.185 - Velocity-time graph for an oscillating system

You can also represent the **velocity** (v) of an oscillating object over time by using a graph.

$$v = -A\omega\sin \omega t$$

As we know that **velocity is the change in displacement over time**, we can draw a **velocity-time graph** by using the above formula or by drawing the **gradient function of the above graph**, noting that the maximum (ωA) and minimum velocity (-ωA)) occurs when x is 0, as expected from the above formula:

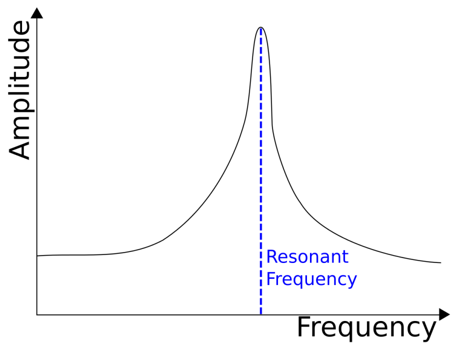


As **acceleration** is the change in velocity over time, you can find the acceleration of the system at a point, by finding the **gradient** of the curve at that point.

13.186 - Resonance

Resonance is where the amplitude of oscillations of a system drastically increases due to gaining an increased amount of energy from the driving force. Resonance has many applications for example:

- **Instruments** - An instrument such as a flute has a long tube in which air resonates, causing a stationary sound wave to be formed.
- **Radio** - These are tuned so that their electric circuit resonates at the same frequency as the desired broadcast frequency.
- **Swing** - If someone pushes you on a swing they are providing a driving frequency, which can cause resonance if it's equal to the resonant frequency and cause you to swing higher.



Resonance occurs when the driving frequency, which is the frequency of the force driving the system, is **equal** to the natural frequency of the system *(the natural frequency is explained in section 13.189)*.

13.188 - Damping and energy conservation

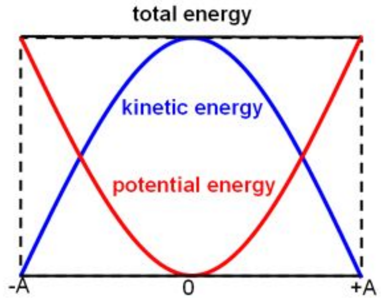
Alongside all the positive consequences of resonance described above, resonance can also have negative consequences, such as causing damage to a structure, for example a **bridge** when the people crossing it are providing a driving frequency close to the natural frequency, will begin to oscillate violently which could be very dangerous and damage the bridge. Therefore **damping can be used to decrease the effect of resonance** - **damping** is where a force acts on an oscillating system and energy is lost from the system to its environment, leading to **reduced amplitude** of oscillations.

An oscillating system **cannot gain or lose energy** unless there are any external forces acting on it - this is the principle of **conservation of energy**.

For any simple harmonic motion system, kinetic energy is transferred to potential energy and back as the system oscillates, the type of potential energy depends on the system.

At the **amplitude** of its oscillations the system will have the **maximum amount of potential energy**, as it moves towards the equilibrium position, this potential energy is converted to kinetic energy so that at the **centre of its oscillations** the **kinetic energy is at a maximum**, then as the system moves away from the equilibrium again, the kinetic energy is transferred to potential energy until it is at a maximum again and this process repeats for one full oscillation.

In an **undamped** system, there are no external forces acting, therefore no energy is lost to the environment and so the total energy of the system remains constant as shown in the diagram below.



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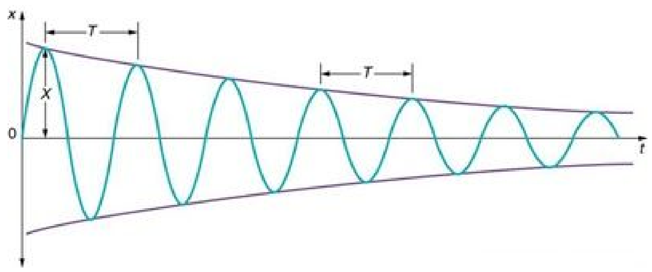
In a **damped** system, there is (at least one) external force acting, therefore some energy will be lost to the environment. For example, in a simple pendulum, air resistance acts against the motion of the pendulum and energy is lost as heat, so over time, the energy of the system decreases.

The diagram below shows the displacement over time in a damped oscillating system.



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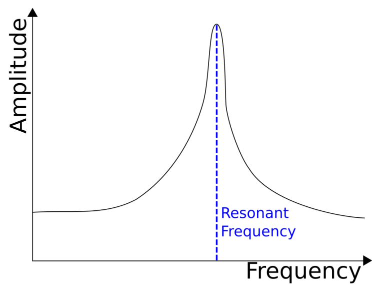
13.189 - Free and forced vibrations

Free vibrations occur when no external force is continuously acting on the system, therefore the system will oscillate at its **natural frequency**.

Forced vibrations are where a system experiences an **external driving force** which causes it to oscillate, the frequency of this driving force, known as driving frequency, is significant.

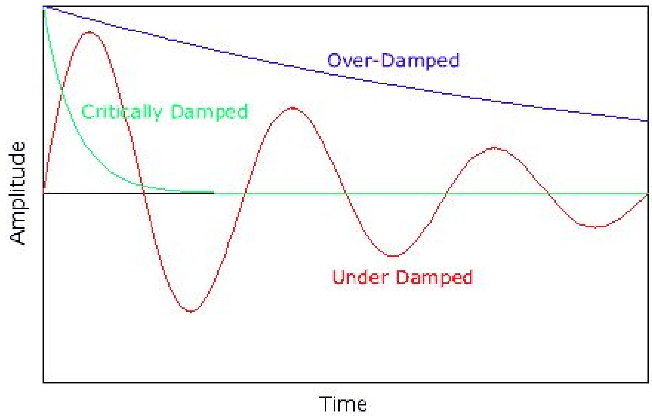
13.190 - The effect of damping on resonance

As mentioned previously, if the **driving frequency is equal to the natural frequency of a system** (also known as the resonant frequency), then **resonance** occurs, meaning the amplitude of forced oscillation increases drastically.

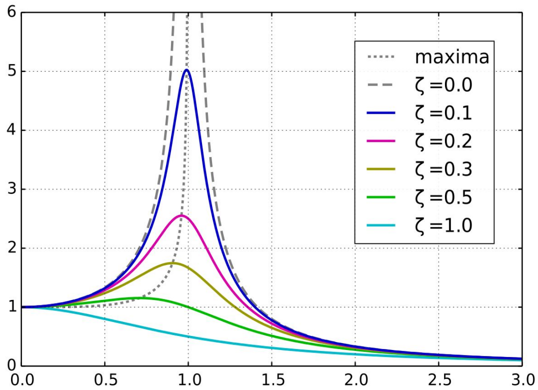


There are 3 main types of damping:

- **Light damping** - This is also known as under-damping and this is where the amplitude gradually decreases by a small amount each oscillation.
- **Critical damping** - This reduces the amplitude to zero in the shortest possible time (without oscillating).
- **Heavy damping** - This is also known as over-damping, and this is where the amplitude reduces slower than with critical damping, but also without any additional oscillations.



Damping can be used to decrease the effect of resonance, different types of damping will have different effects, **as the degree of damping increases**, the resonant frequency decreases (shifts to left on a graph), the maximum amplitude decreases and the peak of maximum amplitude becomes wider, these effects are shown in the graph below, where ζ is the damping ratio, $\zeta = 1$ represents critical damping.



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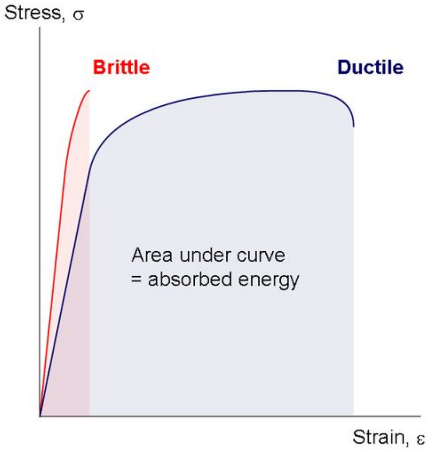
13.191 - Reducing the amplitude of oscillation

In the diagram above, you can see the effects of damping on the amplitude of oscillation of a system: as the degree of **damping increases**, the **amplitude decreases**.

A **ductile** material is one which can undergo a large amount of **plastic deformation** before fracturing, meaning it will be permanently deformed. The plastic deformation of a ductile material can be used to **reduce the amplitude of oscillations**, this happens because **energy is used to**

deform the material, decreasing the kinetic energy of the system and so the amplitude of oscillations decreases.



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For example, a climbing rope is manufactured so that it will reduce the amplitude of oscillations if a climber falls as quickly as possible (this is **critical damping**), meaning that they can stay safe, while not having to bounce many times before stopping (as you would with a bungee cord). As the climbing rope suffers plastic deformation in order to reduce the amplitude of oscillations, it cannot be used a second time after a climber falls with it.

This is just a single example, but many **ductile materials** (such as metals) are used in this way, to **reduce the amplitude of oscillations**.